PHIL 4310: Advanced Logic
Spring 2026
Homework 2

Read Chapter 2.3 of *Logic for Philosophy*

For examples of Fitch style natural deduction systems, see *The Logic Book* or *Language, Proof and Logic*. For a Lemmon-style system, see *The Logic Primer* (all in dropbox folder).

Homework questions:

Part I:

1) Do all of exercise 2.5 (problems a-j)

Part II:

The primitive sequent rules given by Sider form a complete set in the sense that any valid sequent can be proved using just those rules. But some proofs can get very tedious so it is a good thing to be able to use derived uses (shortcuts - like lemmas in a complicated proof). Here are some particularly useful shortcuts (treat all of these sentence letters as metavariables):

Modus Tollens	Disjunctive Syllogism	Biconditionals
$P\rightarrow Q, \sim Q \Rightarrow \sim P$	$P\vee Q, \sim P \Rightarrow Q$	$P\leftrightarrow Q, P \Rightarrow Q$
		$P\leftrightarrow Q, Q \Rightarrow P$
Conditionals	DeMorgan's Laws	$\sim(P\leftrightarrow Q) \Leftrightarrow \sim P\leftrightarrow Q$
$\sim P\vee Q \Leftrightarrow P\rightarrow Q$	$\sim(P\vee Q) \Leftrightarrow \sim P\wedge\sim Q$	$P\leftrightarrow Q, \sim P \Rightarrow \sim Q$
$P\wedge\sim Q \Leftrightarrow \sim(P\rightarrow Q)$	$\sim(P\wedge Q) \Leftrightarrow \sim P\vee\sim Q$	$P\leftrightarrow Q \Leftrightarrow \sim P\leftrightarrow\sim Q$

Prove the following sequents. For these problems, you can use any of Sider's primitive rules or any of the above derived rules.

1. $P\vee Q\ ,\ Q\rightarrow R \ \Rightarrow\ \sim R\rightarrow P$
2. $P\vee Q \ \Rightarrow\ (\sim Q\rightarrow\sim P)\rightarrow Q$
3. $P\rightarrow Q, \sim P\rightarrow R \ \Rightarrow\ Q\vee R$
4. $\sim(P\wedge Q), \sim(\sim P\wedge Q) \ \Rightarrow\ \sim Q$
5. $(P\rightarrow Q)\rightarrow P, P\rightarrow R \ \Rightarrow\ R$
6. $(P\rightarrow Q)\rightarrow Q \ \Rightarrow\ (Q\rightarrow P)\rightarrow P$
7. $P\leftrightarrow(Q\leftrightarrow R) \Rightarrow (\sim Q\wedge\sim P)\rightarrow R$

Part II:

On page 66, Sider mentions one kind of redundancy in his system - if you treat some connectives as parts of abbreviations for others, you could eliminate some of the rules. But there is a more blatant kind of redundancy in the system. The rule 'Add' is strictly redundant in the sense that any sequent that can be proven in Sider's system can be proven without using the rule 'Add' (and with no alterations to the grammar or anything else about the system). Prove this claim.

Part III:

Some inhabitants of the island of knights and knaves are werewolves. Werewolves can be either knights or knaves. You know that exactly one of A,B,C is a werewolf.

A says "C is a werewolf"

B says "I am not a werewolf"

C says "At least two of us are knaves"

1) Is the werewolf a knight or a knave?

2) If you had to travel with one at night, who would you pick?